

Anomalous Triple Gauge Boson Couplings in $e^-e^+ \rightarrow \gamma\gamma$ for Non Commutative Standard Model

N. G. Deshpande^{1,*} and Sumit K. Garg^{2,†}

¹*Institute of Theoretical Science, University of Oregon, Eugene, Oregon 97403, USA*

²*Centre for High Energy Physics, Indian Institute of Science, Bangalore 560 012, India*

Abstract

We investigate $e^+e^- \rightarrow \gamma\gamma$ process within the Seiberg-Witten expanded noncommutative standard model(NCSM) scenario in the presence of anomalous triple gauge boson couplings. This study is done with and without initial beam polarization and we restrict ourselves to leading order effects of non commutativity i.e. $O(\Theta)$. The non commutative(NC) corrections are sensitive to the electric component($\vec{\Theta}_E$) of NC parameter. We include the effects of earth rotation in our analysis. This study is done by investigating the effects of non commutativity on different time averaged cross section observables. We have also defined forward backward asymmetries which will be exclusively sensitive to anomalous couplings. We have looked into the sensitivity of these couplings at future experiments at the International Linear Collider(ILC). This analysis is done under realistic ILC conditions with the Center of mass energy(c.m.) $\sqrt{s} = 800\text{GeV}$ and integrated luminosity $L=500\text{fb}^{-1}$. The scale of non commutativity is assumed to be $\Lambda = 1\text{TeV}$. The limits on anomalous couplings of the order 10^{-1} from forward backward asymmetries while much stringent limits of the order 10^{-2} from total cross section are obtained if no signal beyond SM is seen.

*Electronic address: desh@uoregon.edu

†Electronic address: sumit@cts.iisc.ernet.in

I. INTRODUCTION

Triple gauge boson couplings arise in the Standard Model(SM) due to the non Abelian nature of the theory and thus gives the possibility of exploring the bosonic sector of the SM. Many of such couplings do not appear in SM at the tree level and even at higher orders thus they are expected to be very small. Hence the precise measurement of these couplings could indicate a signal for new physics beyond the SM even if there is no direct production of particles beyond SM spectrum. A linear collider at a center of mass energy of 800 GeV or more in high luminosity regime provide a unique opportunity to measure such couplings with an unprecedented accuracy where we can distinguish such effects from the SM predictions. Moreover the availability of initial beam polarization option, can significantly enhance the sensitivity to such effects.

In this study we investigated the expected sensitivity of triple gauge boson couplings $Z\gamma\gamma$ and $\gamma\gamma\gamma$, to leading order that will contribute in $e^-e^+ \rightarrow \gamma\gamma$ process at proposed International Linear Collider(ILC) [1, 2]. This investigation is done within the frame work of non commutative SM(NCSM).

Quantum field theories constructed on non commutative(NC) space time have been extensively explored in the past few years. This field has received much attention due to its possible connection with quantum gravity and because of its natural origin in string theories. Infact Seiberg and Witten[3] described how NC gauge theory can emerge as a low energy manifestation of open string theory. This work has stimulated many paper on non commutative models[4–9].

Hence keeping in mind the above considerations it is reasonable to investigate field theories, and in particular the standard model of particle physics on non commutative space time. Here we adopt an approach based on Seiberg-Witten Map(SWM) popularized by the Munich group[10–18].

In this approach, to construct the NC extension of the standard model (SM) [13, 14, 17, 18], which uses the same gauge group and particle content one expands the NC gauge fields in non linear power series of Θ [3, 11, 12]. At face value it can be seen from the above map that SW approach leads to a field theory with an infinite number of vertices and Feynman graphs thereby leading to an uncontrolled degree of divergence inturn giving an impression of complete failure of perturbative renormalization. But over the years a

number of studies have shown that it is possible to construct anomaly free, renormalizable, and effective theories at one loop and first order in Θ [19–27]. The above mentioned studies provide confidence in using the NC SW expanded SM for phenomenological purposes. However it should be mentioned here that the celebrated IR/UV mixing does not exist in the above Θ expanded approach. Though this is not a drawback in the scales of our interest there do exist certain phenomena that require all orders of the NC parameter be retained. This led to the so called Θ -exact approach, that is from the exact solutions of the SW equations. The phenomenological consequences of this have been explored in [28–31].

The reason why NC collider phenomenology is interesting, comes from the fact that the scale of non commutativity can be as low as a few TeV[14, 28–35], which is amenable for exploration at the present or the future colliders. This has led to a great deal of interest in phenomenology of the NCSM with SWM. Many phenomenological signatures have been studied by different research groups. These works were mainly done [32, 33, 36–53] with unpolarized beams with leading corrections to SM starting from $O(\Theta^2)$. However few studies [16, 54–56] are also done with corrections at the $O(\Theta)$ in cross section. Some previous studies[14, 16, 32, 44, 54, 56] have also looked into the sensitivity of anomalous triple gauge couplings at Large Hadron Collider(LHC) and ILC.

In this work we have calculated ($O(\Theta)$) corrections for pair annihilation with and without initial beam polarization. Here, unlike NCQED case, non commutative effects at leading order also appear in unpolarized cross section due to the presence of axial vector coupling of the Z boson. We have also taken into account the effect of earth’s rotation[57–60] on observable signals of non commutativity. The effects of Non commutativity is studied on various time averaged observables to check the sensitivity of anomalous couplings. Here we note that this process has been studied previously[61] with unpolarized initial beams and polarized final states. Note that observation of final state polarization of photons is not possible at the high energies of ILC.

We have looked at the sensitivity of the anomalous couplings at the International Linear Collider(ILC)[1, 2] with realistic beam luminosity of 500 fb^{-1} . The availability of longitudinal beam polarization of one or both of the e^-e^+ beams, can give the opportunity to test the couplings which otherwise are absent in the observables with unpolarized initial beam. If these beams become available at future Linear Colliders then it will serve as crucial test for these anomalous couplings of NCSM in the process we discuss.

The rest of the paper is organized as follows. In section II, we give the calculational details of our work for the mentioned process. In section III, we will present our numerical results. Finally we conclude with a section on summary of our results.

II. CROSS SECTIONS IN THE LABORATORY FRAME

This process in NCQED proceed at the tree level by the following diagrams(Fig.1). The first two diagrams also appears in pure QED while 3rd one arises just because of non commutative nature of space time and is a contact interaction.

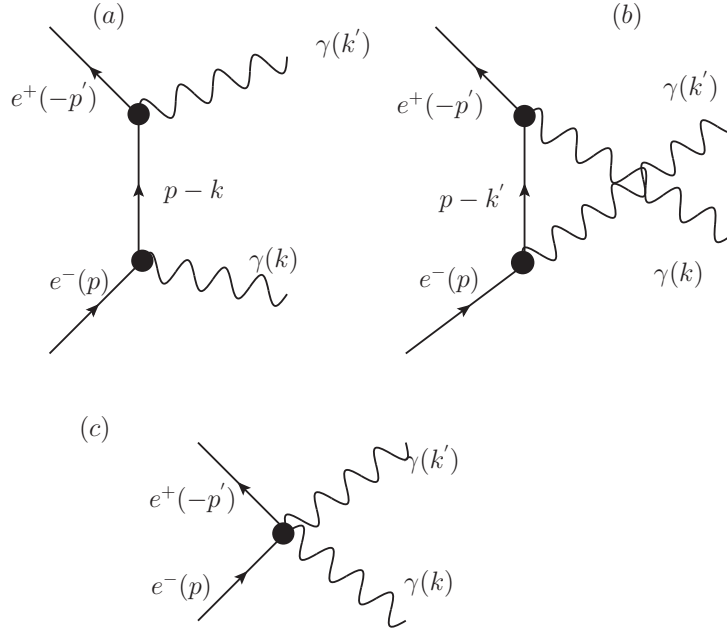


FIG. 1: **NCQED Diagrams.** Feynman Diagrams corresponding to NCQED.

However in non minimal version of Standard model this process also contain two additional s-channel diagrams(Fig. 2) with anomalous triple gauge boson vertices $Z\gamma\gamma, \gamma\gamma\gamma$.

The squared amplitude for the above process is given by the expression

$$|A|^2 = |A_{SM}|^2 + (A_{SM})^* A_{NC}[O(\Theta)] + A_{SM}(A_{NC})^*[O(\Theta)] + |A_{NC}|^2[O(\Theta^2)]$$

Here as mentioned in introduction we are restricting ourselves only to $O(\Theta)$ thus the interference between SM and NC term can provide required corrections to cross section.

Since non commutative parameter is considered as elementary constant in nature so its direction is fixed in some non rotating coordinate system(can be taken to celestial sphere).

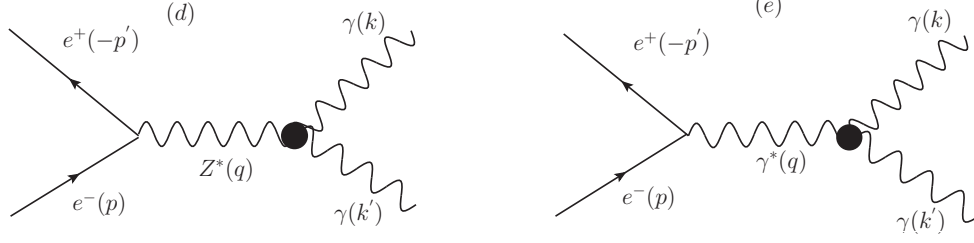


FIG. 2: **Non Minimal NCSM:** s-channel Feynman Diagrams with anomalous couplings. Here $q = k + k' = p + p'$ is the momentum of the propagator.

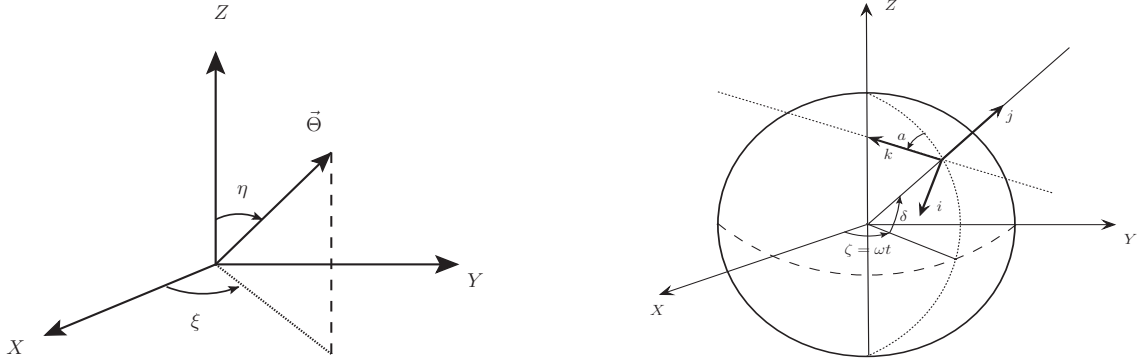


FIG. 3: X-Y-Z is the primary coordinate system while $\{\hat{i} - \hat{j} - \hat{k}\}$ are unit vectors pertaining to the laboratory coordinate system. The direction of $\vec{\Theta}$ is defined by angles η and ξ .

However the experiment is done in laboratory coordinate system which is rotating with earth's rotation. So one should take into account these rotation effects on $\Theta_{\mu\nu}$ in this frame before moving towards the phenomenological investigations.

These effects were considered in many previous studies[57–60] but we are here following the lines of[59]. In the laboratory coordinate system, the orthonormal basis of the non rotating(primary) coordinate system can be written as

$$\vec{i} = \begin{pmatrix} c_a s_\zeta + s_\delta s_a c_\zeta \\ c_\delta c_\zeta \\ s_a s_\zeta - s_\delta c_a c_\zeta \end{pmatrix}, \quad \vec{j} = \begin{pmatrix} -c_a c_\zeta + s_\delta s_a s_\zeta \\ c_\delta s_\zeta \\ -s_a c_\zeta - s_\delta c_a s_\zeta \end{pmatrix}, \quad \vec{k} = \begin{pmatrix} -c_\delta s_a \\ s_\delta \\ c_\delta c_a \end{pmatrix}. \quad (1)$$

Here we have used the abberivations $c_\alpha = \cos \alpha$, $s_\alpha = \sin \alpha$ etc. (δ, a) defines the location of experiment with $-\pi/2 \leq \delta \leq \pi/2$ and $0 \leq a \leq 2\pi$. More details can be found in Ref. [59].

Thus the NC parameter in the Laboratory frame is given by electric and magnetic com-

ponents

$$\vec{\Theta}_E = \Theta_E(\sin \eta_E \cos \xi_E \vec{i} + \sin \eta_E \sin \xi_E \vec{j} + \cos \eta_E \vec{k}) \quad (2)$$

with

$$\vec{\Theta}_E = (\Theta^{01}, \Theta^{02}, \Theta^{03}) \quad \Theta_E = |\vec{\Theta}_E| = 1/\Lambda_E^2$$

Here (η, ξ) specifies the direction of NC parameter $(\Theta_{\mu\nu})$ w.r.t primary coordinates system. Θ_E is the absolute values of its electric components with corresponding scale Λ_E .

Our results are based on Feynman rules for NCSM given in Ref. [17, 18]. For evaluating cross section we have used Standard Trace technique and various traces are obtained by the Mathematica Package FeynCalc[62]. The trace results are also cross checked in Symbolic Manipulation programme FORM[63].

Thus in the Center of Mass frame $(A(p) + B(p') \rightarrow A(k) + B(k'))$

$$\begin{aligned} p^\mu &= \frac{\sqrt{s}}{2} \{1, 0, 0, 1\} \\ p^{\mu'} &= \frac{\sqrt{s}}{2} \{1, 0, 0, -1\} \\ k^\mu &= \frac{\sqrt{s}}{2} \{1, \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta\} \\ k^{\mu'} &= \frac{\sqrt{s}}{2} \{1, -\sin \theta \cos \phi, -\sin \theta \sin \phi, -\cos \theta\} \end{aligned} \quad (3)$$

where θ is the polar angle and ϕ is the azimuthal angle, with initial beam direction chosen as the z-axis.

Due to the breaking of Lorentz invariance for fixed Θ background, non commutativity of space time lead to dependence of cross section on azimuthal angle which is absent in Standard Model. The final cross section formulae for different cases are given by:

A. Unpolarized Case

The differential cross section for e^-e^+ unpolarized case is given by:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\vec{\Theta}_E} = \frac{\alpha^2}{s} \left[(1 + \cos^2 \theta) \csc^2 \theta + \bar{s}_E \{ L_1^\theta (\Theta^{02} \cos \phi - \Theta^{01} \sin \phi) \} \right] , \quad (4)$$

$$L_1^\theta = 2C_1 \csc \theta$$

$$C_1 = \frac{C_A}{(1 - M_Z^2/s)} K_{Z\gamma\gamma} \quad (5)$$

$$(6)$$

B. Polarized Case

The differential cross section for e^- in Right polarized state is given by:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\vec{\Theta}_E} = \frac{\alpha^2}{s} \left[(1 + \cos^2 \theta) \csc^2 \theta + \bar{s}_E \{ M_1^\theta (\Theta^{02} \cos \phi - \Theta^{01} \sin \phi) \} \right] , \quad (7)$$

$$M_1^\theta = 2C_2 \csc \theta - \cot \theta$$

$$C_2 = \left[\frac{(C_A - C_V)}{(1 - M_Z^2/s)} K_{Z\gamma\gamma} - K_{\gamma\gamma\gamma} \sin 2\theta_W \right] \quad (8)$$

$$(9)$$

The differential cross section for e^- in Left polarized state is given by:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\vec{\Theta}_E} = \frac{\alpha^2}{s} \left[(1 + \cos^2 \theta) \csc^2 \theta + \bar{s}_E \{ N_1^\theta (\Theta^{02} \cos \phi - \Theta^{01} \sin \phi) \} \right] , \quad (10)$$

$$N_1^\theta = 2C_3 \csc \theta + \cot \theta$$

$$C_3 = \left[\frac{(C_A + C_V)}{(1 - M_Z^2/s)} K_{Z\gamma\gamma} + K_{\gamma\gamma\gamma} \sin 2\theta_W \right] \quad (11)$$

$$(12)$$

C_V and C_A are the vector and axial vector coupling of Z boson with electron and are given by $(-1 + 4 \sin^2 \theta_W)/2$ and $-1/2$ respectively. Since it is difficult to get time dependent data so one have to take average over full day to be compared with the experiment. So we will consider here following cross section observables to reveal the effects of non commutativity

$$\left\langle \frac{d\sigma}{d\phi} \right\rangle_T \equiv \frac{1}{T_{day}} \int_0^{T_{day}} \frac{d\sigma}{d\phi} dt, \quad (13)$$

$$\langle \sigma \rangle_T \equiv \frac{1}{T_{day}} \int_0^{T_{day}} \sigma dt, \quad (14)$$

where

$$\frac{d\sigma}{d\phi} \equiv \int_{-1}^1 d(\cos \theta) \frac{d\sigma}{d \cos \theta d\phi}, \quad (15)$$

$$\sigma \equiv \int_{-1}^1 d(\cos \theta) \int_0^{2\pi} d\phi \frac{d\sigma}{d \cos \theta d\phi}. \quad (16)$$

In addition to these observables, since the terms containing anomalous couplings in cross section flip sign under transformation $(\theta \rightarrow (\pi - \theta), \phi \rightarrow (\pi + \phi))$, so we can define following forward backward asymmetry (with appropriate cuts on azimuthal angle(ϕ)) which will only be sensitive to these couplings.

$$A_{FB}^U(\theta_0) = \frac{\langle \sigma_F^f(\theta_0) \rangle_T - \langle \sigma_B^f(\theta_0) \rangle_T}{\sigma_{tot}(\theta_0)} \quad (17)$$

$$= \frac{C_1(4\theta_0 - \pi) \bar{s}_E \sin a \cos \delta \cos \eta}{\pi(\cos \theta_0 + 2 \log[\tan \frac{\theta_0}{2}])} \quad (18)$$

where

$$\langle \sigma_F^f(\theta_0) \rangle_T = \frac{1}{T_{day}} \int_0^{T_{day}} \left[\int_{\theta_0}^{\pi/2 - \theta_0} \int_{\phi=0}^{\pi} \frac{d\sigma}{d\Omega} \sin \theta d\theta d\phi \right] dt \quad (19)$$

$$\langle \sigma_B^f(\theta_0) \rangle_T = \frac{1}{T_{day}} \int_0^{T_{day}} \left[\int_{\pi/2+\theta_0}^{\pi-\theta_0} \int_{\phi=\pi}^{2\pi} \frac{d\sigma}{d\Omega} \sin \theta d\theta d\phi \right] dt \quad (20)$$

$$\sigma_{tot}(\theta_0) = \int_{\theta_0}^{\pi-\theta_0} \int_{\phi=0}^{2\pi} \frac{d\sigma}{d\Omega} \sin \theta d\theta d\phi \quad (21)$$

Similarly one can also define polarized forward backward asymmetry

$$A_{FB}^P(\theta_0) = \frac{\langle \sigma_{FL}^f(\theta_0) \rangle_T - \langle \sigma_{BL}^f(\theta_0) \rangle_T - (\langle \sigma_{FR}^f(\theta_0) \rangle_T - \langle \sigma_{BR}^f(\theta_0) \rangle_T)}{\sigma_{tot}(\theta_0)} \quad (22)$$

$$= \frac{(C_2 - C_3)(\pi - 4\theta_0)\bar{s}_E \sin a \cos \delta \cos \eta}{\pi(\cos \theta_0 + 2 \log[\tan \frac{\theta_0}{2}])} \quad (23)$$

where

$$\langle \sigma_{FL}^f(\theta_0) \rangle_T = \frac{1}{T_{day}} \int_0^{T_{day}} \left[\int_{\theta_0}^{\pi/2-\theta_0} \int_{\phi=0}^{\pi} \frac{d\sigma}{d\Omega} \sin \theta d\theta d\phi \right] dt \quad (24)$$

$$\langle \sigma_{BL}^f(\theta_0) \rangle_T = \frac{1}{T_{day}} \int_0^{T_{day}} \left[\int_{\pi/2+\theta_0}^{\pi-\theta_0} \int_{\phi=\pi}^{2\pi} \frac{d\sigma}{d\Omega} \sin \theta d\theta d\phi \right] dt \quad (25)$$

$$(26)$$

where L and R refer to the helicity of the incident electron beam; F and B stand for forward and backward. For unpolarized case, the coefficient of $K_{Z\gamma\gamma}$ in forward backward asymmetry is smaller by a factor of $2C_V/C_A$ then to its polarized counterpart so the corresponding limit for $K_{Z\gamma\gamma}$ will be stringent in this case.

III. NUMERICAL ANALYSIS

In this section we will provide the numerical results of our investigation. In order to check the sensitivity of anomalous couplings($K_{Z\gamma\gamma}, K_{\gamma\gamma\gamma}$), we studied previously defined forward backward asymmetries(A_{FB}) and total cross section($\langle \sigma \rangle_T$). We fixed the initial beam energy at $\sqrt{s}(= E_{com}) = 800$ GeV. The position of Lab system is fixed by taking $\delta = \pi/4$ and

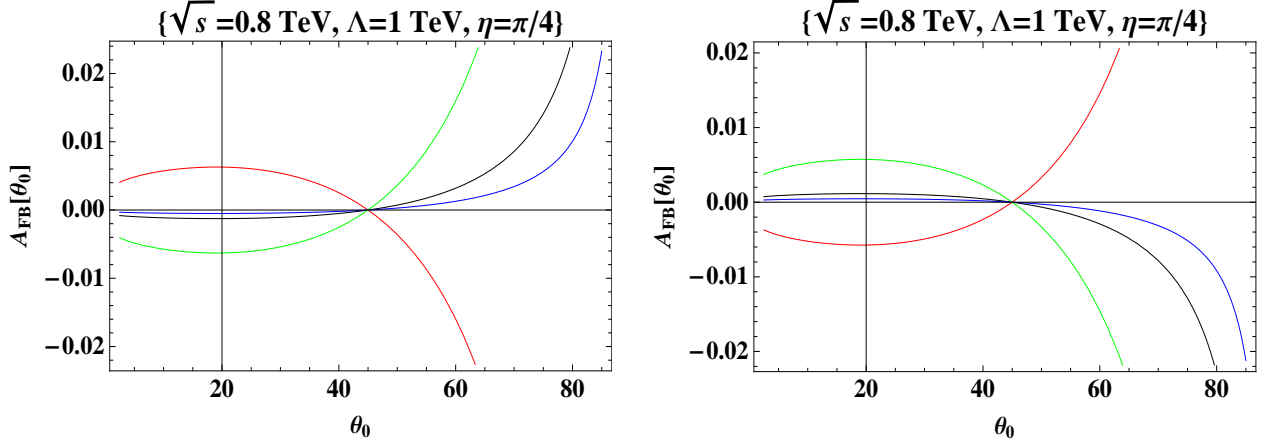


FIG. 4: Forward backward asymmetry for Unpolarized case(Left) and for Polarized case(Right)vs polar cutoff angle(θ_0). Red, Black, Blue and Green curves corresponds to $K_{Z\gamma\gamma} = -0.25, 0.05, 0.02, 0.25$ in unpolarized case and $K_{\gamma\gamma\gamma} = -0.25, 0.05, 0.02, 0.25$ (with $K_{Z\gamma\gamma} = 0$) for polarized case respectively.

$a = \pi/4$. For our sensitivity analysis, we assume an integrated luminosity of $L=500 \text{ fb}^{-1}$ and we have fixed the NC parameters at $\eta = \pi/4$ and $\Lambda = 1000 \text{ GeV}$ while initial phase(ξ) dependence disappears in time averaged observables.

Here for studying total cross section we applied a cut of $(0 - \pi)$ on azimuthal angle ϕ since non commutative effects disappear once we integrate over the full azimuthal angle $(0 - 2\pi)$. Our results are useful for case $s/\Lambda^2 < 1$ since in this domain one can safely ignore higher order corrections to cross section.

For deriving limits on anomalous couplings we will make use of expressions for forward backward asymmetry along with total cross section. Fig. 4 shows variation of previously defined forward backward asymmetries plotted against polar cutoff angle θ_0 for different values of anomalous couplings. As evident from the plots, the magnitude of asymmetries become larger for higher values of polar cutoff angle(θ_0). Thus using higher θ_0 will give better limits on couplings.

The asymmetries are then used to calculate 90% CL limits with realistic integrated luminosities in the absence of any signal at ILC. The limit on the coupling at a polar cut off

angle(θ_0) is related to the value of $A(\theta_0)$ of the asymmetry by[64, 65]:

$$\lambda^{lim}(\theta_0) = \frac{1.64}{|A(\theta_0)|\sqrt{\sigma_{tot}(\theta_0) \cdot L}} \quad (27)$$

where $|A(\theta_0)|$ is the absolute value of the asymmetry for unit value of the coupling.

From Eq. 18 we see that A_{FB}^U solely depends on $K_{Z\gamma\gamma}$, therefore an independent limit can be placed on it. However A_{FB}^P depends on $K_{Z\gamma\gamma}$ as well as on $K_{\gamma\gamma\gamma}$. Thus for evaluating limit on $K_{\gamma\gamma\gamma}$ we have assumed other anomalous coupling to be zero. From Figs. 6 it is clear that the best limit for $|K_{Z\gamma\gamma}|$ and $|K_{\gamma\gamma\gamma}|$ is achieved for an cutoff angle $\theta_0 = 75^\circ$.

Following the same procedure one can obtain limits from total time averaged cross section. In Fig. 5 we have plotted the total time averaged cross section for different values of anomalous couplings. Here unlike asymmetries significant deviation from SM case is obtained for smaller values of cut-off angle(θ_0). We have also derived the limits on these couplings in case of no excess in signal events is observed at ILC.

However here the limits are obtained by using the condition that the excess number of events beyond expected from SM should be smaller than the statistical error in the number of SM events. This translates to $L|\sigma_{NP}(\theta_0)| < 1.64\sqrt{\sigma_{tot}(\theta_0) \cdot L}$ where σ_{NP} is the NC contribution to the total cross section and L is the integrated luminosity which we assumed to be 500 fb^{-1} for current study.

From Eqs.4, 7 we see that σ_T for unpolarized case solely depends on $K_{Z\gamma\gamma}$, therefore an independent limit can be placed on it. However σ_T for unpolarized case depends on $K_{Z\gamma\gamma}$ as well as on $K_{\gamma\gamma\gamma}$. Thus for evaluating limit on $K_{\gamma\gamma\gamma}$ we have assumed other anomalous coupling to be zero. From Figure 7 it is clear that the best limit for $|K_{Z\gamma\gamma}|$ and $|K_{\gamma\gamma\gamma}|$ is obtained for an cutoff angle $\theta_0 = 30^\circ$.

Coupling	Limits-Unpolarized case		Limits-Polarized case	
	$\langle\sigma\rangle_T$	A_{FB}	$\langle\sigma\rangle_T$	A_{FB}
$ K_{Z\gamma\gamma} $	4.2×10^{-2}	1.4×10^{-1}	4.5×10^{-2}	9.2×10^{-1}
$ K_{\gamma\gamma\gamma} $	-	-	9.2×10^{-2}	1.5×10^{-1}

TABLE I: 90 % CL limits on the couplings from $\langle\sigma\rangle_T$ for a cut-off angle of 32° and A_{FB} for a cut-off angle of 75° . These limits are derived for $\sqrt{s} = 800 \text{ GeV}$, $\Lambda = 1000 \text{ GeV}$ and integrated luminosity, $L = 500 \text{ fb}^{-1}$.

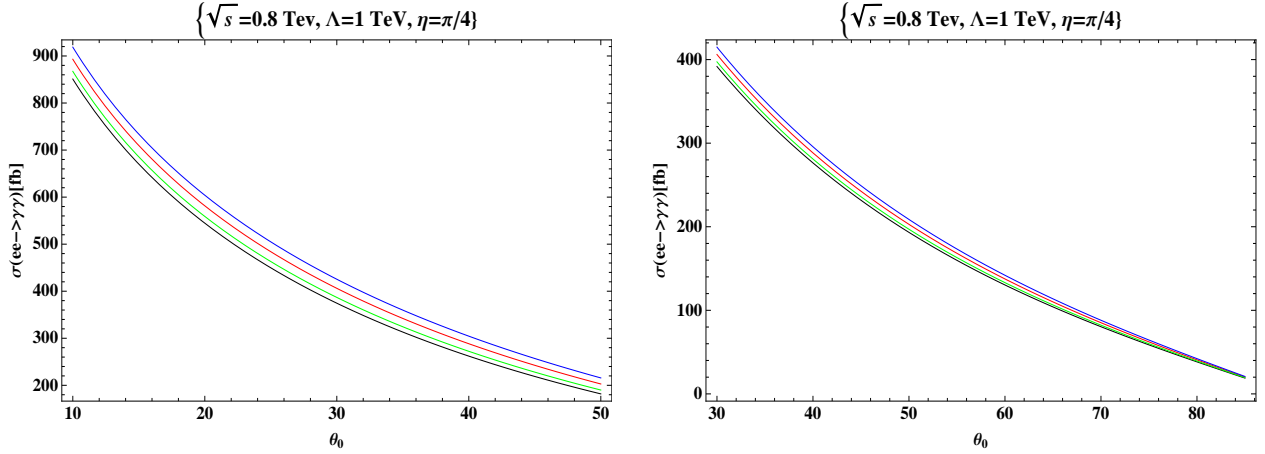


FIG. 5: Time averaged total cross section for Unpolarized case with different values of K_{Zgg} (Left) and for $K_{\gamma\gamma\gamma}$ (Right). Red, Black, Green and Blue curves corresponds to $K_{Z\gamma\gamma} = 0.0, 0.9, 0.55, -0.55$ in unpolarized case and $K_{\gamma\gamma\gamma} = 0.0, 0.9, 0.55, -0.55$ (with $K_{Z\gamma\gamma} = 0$) for polarized case respectively.

In Table I we have quoted the derived limits from asymmetries as well as from time averaged total cross section. The asymmetries give a limit of order 10^{-1} while from total cross section limits are much more stringent of the order of 10^{-2} . Thus total cross section is proven to be much useful observable then forward backward asymmetries.

IV. SUMMARY

The extension of SM to NC space time with motivations coming from string theory and quantum gravity provides interesting phenomenological implications since scale of non commutativity could be as low as a few TeV, which can be explored at the present or future colliders. In the present work we focused on exploring the sensitivity of anomalous couplings ($K_{Z\gamma\gamma}, K_{\gamma\gamma\gamma}$) that will contribute to the process $e^+e^- \rightarrow \gamma\gamma$ process at ILC.

We have done our study with unpolarized as well as taking into account the initial beam polarization effects. We restricted ourselves to the leading order effects of non commutativity to be occurred at leading order in Θ (i.e. $O(\Theta)$) at cross section level. Unlike NCQED case non commutative effects at $O(\Theta)$ also appear in unpolarized cross section due to the presence

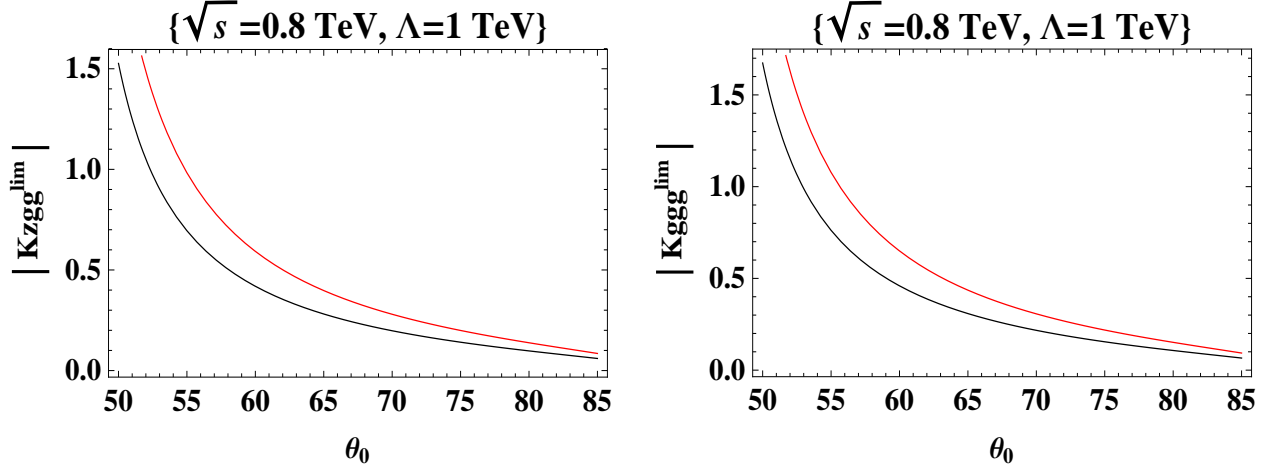


FIG. 6: Limit on $K_{Z\gamma\gamma}$ (Left) from unpolarized forward backward asymmetry and on $K_{\gamma\gamma\gamma}$ (Right) from polarized forward backward asymmetry. Black and Red curve corresponds to $\eta = \pi/4$ and $\eta = \pi/3$ respectively.

of axial vector coupling of Z boson.

In this analysis we have also taken into account the apparent time variation of non commutative parameter($\Theta^{\mu\nu}$) in Laboratory frame. We have used time averaged observables for this study.

The NC corrections to the considered process are sensitive to the electric component($\vec{\Theta}_E$) of NC parameter ($\vec{\Theta}$). However for checking the sensitivity of anomalous couplings at ILC we used time averaged total cross section and forward backward asymmetries as observables. This analysis is done under realistic ILC conditions with the Center of mass energy(c.m.) $\sqrt{s} = 800\text{GeV}$ and integrated luminosity $L=500\text{fb}^{-1}$. The scale of non commutativity is assumed to be $\Lambda = 1\text{TeV}$ and Lab coordinates are fixed to be $(\delta, a) = (\pi/4, \pi/4)$.

The observables for unpolarized case are only sensitive to $K_{Z\gamma\gamma}$ while for polarized case they are sensitive to both couplings. For putting limits from polarized case we have assumed one coupling to be zero at a time. The asymmetries give a limit of order 10^{-1} while limits from total cross section are much more stringent, of the order of 10^{-2} on absolute value of the anomalous couplings.

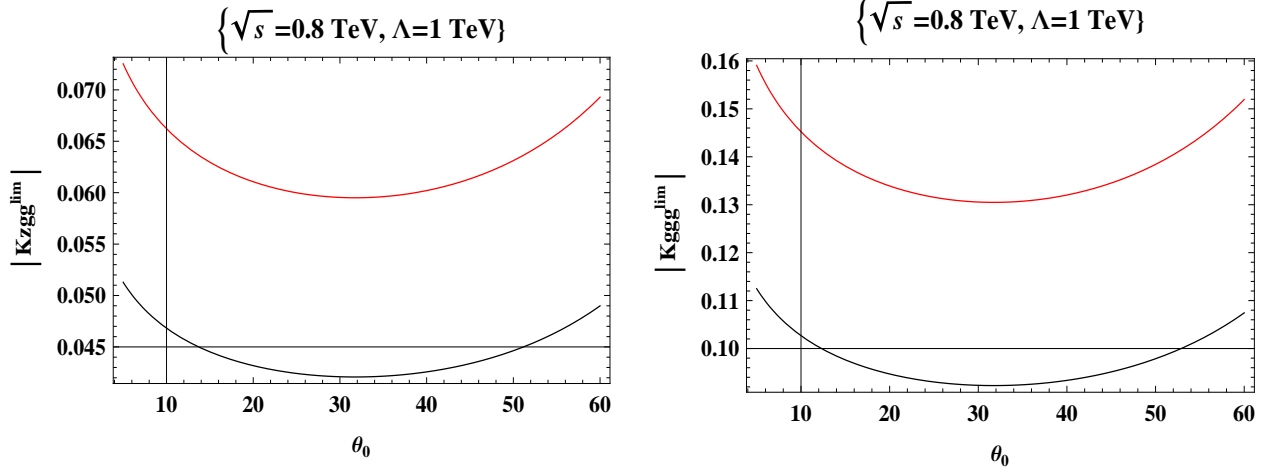


FIG. 7: Limit on $K_{Z\gamma\gamma}$ (Left) from unpolarized total cross section and on $K_{\gamma\gamma\gamma}$ (Right) from polarized cross section. Black and Red curve corresponds to $\eta = \pi/4$ and $\eta = \pi/3$ respectively.

Acknowledgments

S.K.G is grateful to Dr. Rolf Mertig for useful correspondence. The work of N.G.D is supported by the US DOE under Grant No. DE-FG02-96ER40969. S.K.G acknowledges partial support from DST Ramanujan Fellowship SR/S2/RJN-25/2008.

-
- [1] J. (.). Brau *et al.* [ILC Collaboration], arXiv:0712.1950 [physics.acc-ph].
 - [2] G. Aarons *et al.* [ILC Collaboration], arXiv:0709.1893 [hep-ph].
 - [3] N. Seiberg and E. Witten, JHEP **9909**, 032 (1999) [arXiv:hep-th/9908142].
 - [4] H. S. Snyder, Phys. Rev. **71**, 38 (1947).
 - [5] S. Minwalla, M. Van Raamsdonk and N. Seiberg, JHEP **0002**, 020 (2000) [arXiv:hep-th/9912072].
 - [6] A. Matusis, L. Susskind and N. Toumbas, JHEP **0012**, 002 (2000) [arXiv:hep-th/0002075].
 - [7] L. Alvarez-Gaume and M. A. Vazquez-Mozo, Nucl. Phys. B **668**, 293 (2003) [arXiv:hep-th/0305093].
 - [8] J. Jaeckel, V. V. Khoze and A. Ringwald, JHEP **0602**, 028 (2006) [arXiv:hep-ph/0508075].

- [9] S. A. Abel, J. Jaeckel, V. V. Khoze and A. Ringwald, JHEP **0609**, 074 (2006) [arXiv:hep-ph/0607188].
- [10] R. Horvat and J. Trampetic, JHEP **1101**, 112 (2011) [arXiv:1009.2933 [hep-ph]].
- [11] B. Jurco, S. Schraml, P. Schupp and J. Wess, Eur. Phys. J. C **17**, 521 (2000) [arXiv:hep-th/0006246].
- [12] B. Jurco, L. Moller, S. Schraml, P. Schupp and J. Wess, Eur. Phys. J. C **21**, 383 (2001) [arXiv:hep-th/0104153].
- [13] X. Calmet, B. Jurco, P. Schupp, J. Wess and M. Wohlgenannt, Eur. Phys. J. C **23**, 363 (2002) [arXiv:hep-ph/0111115].
- [14] W. Behr, N. G. Deshpande, G. Duplancic, P. Schupp, J. Trampetic and J. Wess, Eur. Phys. J. C **29**, 441 (2003) [arXiv:hep-ph/0202121].
- [15] G. Duplancic, P. Schupp and J. Trampetic, Eur. Phys. J. C **32**, 141 (2003) [arXiv:hep-ph/0309138].
- [16] T. Ohl and J. Reuter, Phys. Rev. D **70**, 076007 (2004) [arXiv:hep-ph/0406098].
- [17] B. Melic, K. Passek-Kumericki, J. Trampetic, P. Schupp and M. Wohlgenannt, Eur. Phys. J. C **42**, 483 (2005) [arXiv:hep-ph/0502249].
- [18] B. Melic, K. Passek-Kumericki, J. Trampetic, P. Schupp and M. Wohlgenannt, Eur. Phys. J. C **42**, 499 (2005) [arXiv:hep-ph/0503064].
- [19] A. Bichl, J. Grimstrup, H. Grosse, L. Popp, M. Schweda and R. Wulkenhaar, JHEP **0106**, 013 (2001) [arXiv:hep-th/0104097].
- [20] C. P. Martin, Nucl. Phys. B **652**, 72 (2003) [arXiv:hep-th/0211164].
- [21] C. P. Martin and C. Tamarit, Phys. Rev. D **72**, 085008 (2005) [arXiv:hep-th/0503139].
- [22] M. Buric, V. Radovanovic and J. Trampetic, JHEP **0703**, 030 (2007) [arXiv:hep-th/0609073].
- [23] D. Latas, V. Radovanovic and J. Trampetic, Phys. Rev. D **76**, 085006 (2007) [arXiv:hep-th/0703018].
- [24] M. Buric, D. Latas, V. Radovanovic and J. Trampetic, Phys. Rev. D **77**, 045031 (2008) [arXiv:0711.0887 [hep-th]].
- [25] C. P. Martin and C. Tamarit, Phys. Lett. B **658**, 170 (2008) [arXiv:0706.4052 [hep-th]].
- [26] M. M. Ettefaghi, M. Haghighat and R. Mohammadi, Phys.Rev. **D 82**, 105017 (2010).
- [27] M. Buric, D. Latas, V. Radovanovic and J. Trampetic, Phys. Rev. D **83**, 045023 (2011) [arXiv:1009.4603 [hep-th]].

- [28] R. Horvat, D. Kekez and J. Trampetic, Phys. Rev. D **83**, 065013 (2011) [arXiv:1005.3209 [hep-ph]].
- [29] R. Horvat, D. Kekez, P. Schupp, J. Trampetic and J. You, arXiv:1103.3383 [hep-ph].
- [30] R. Horvat and J. Trampetic, arXiv:1111.6436 [hep-ph].
- [31] R. Horvat and J. Trampetic, Phys. Rev. D **79**, 087701 (2009) [arXiv:0901.4253 [hep-ph]].
- [32] M. Buric, D. Latas, V. Radovanovic and J. Trampetic, Phys. Rev. D **75**, 097701 (2007).
- [33] G. Abbiendi *et al.* [OPAL Collaboration], Phys. Lett. B **568**, 181 (2003) [arXiv:hep-ex/0303035].
- [34] M. Chaichian, M. M. Sheikh-Jabbari and A. Tureanu, Phys. Rev. Lett. **86**, 2716 (2001) [arXiv:hep-th/0010175].
- [35] S. M. Carroll, J. A. Harvey, V. A. Kostelecky, C. D. Lane and T. Okamoto, Phys. Rev. Lett. **87**, 141601 (2001) [arXiv:hep-th/0105082].
- [36] P. K. Das, N. G. Deshpande and G. Rajasekaran, Phys. Rev. D **77**, 035010 (2008) [arXiv:0710.4608 [hep-ph]].
- [37] J. Hewett, F. J. Petriello and T. G. Rizzo, hep-ph/0201275; J. Hewett *et al.*, Phys. Rev. D **64**, 075012 (2001), Phys. Rev. D **66**, 036001 (2002).
- [38] P. Schupp, J. Trampetic, J. Wess and G. Raffelt, Eur. Phys. J. C **36**, 405 (2004) [arXiv:hep-ph/0212292].
- [39] T. Ohl and C. Speckner, Phys. Rev. D **82**, 116011 (2010) [arXiv:1008.4710 [hep-ph]].
- [40] M. Haghighat, M. M. Ettefaghi and M. Zeinali, Phys. Rev. D **73**, 013007 (2006) [arXiv:hep-ph/0511042].
- [41] M. Mohammadi Najafabadi, Phys. Rev. D **74**, 025021 (2006) [arXiv:hep-ph/0606017].
- [42] N. Mahajan, Phys. Rev. D **68**, 095001 (2003) [arXiv:hep-ph/0304235].
- [43] E. O. Iltan, Phys. Rev. D **66**, 034011 (2002) [arXiv:hep-ph/0204332].
- [44] N. G. Deshpande and X. G. He, Phys. Lett. B **533**, 116 (2002) [arXiv:hep-ph/0112320].
- [45] A. Devoto, S. Di Chiara and W. W. Repko, Phys. Rev. D **72**, 056006 (2005) [hep-ph/0503024].
- [46] M. M. Najafabadi, Phys. Rev. D **77**, 116011 (2008) [arXiv:0803.2340 [hep-ph]].
- [47] J. Trampetic, Fortsch. Phys. **56**, 521 (2008) [arXiv:0802.2030 [hep-ph]].
- [48] B. Melic, K. Passek-Kumericki and J. Trampetic, Phys. Rev. D **72**, 054004 (2005) [arXiv:hep-ph/0503133].
- [49] B. Melic, K. Passek-Kumericki and J. Trampetic, Phys. Rev. D **72**, 057502 (2005)

- [arXiv:hep-ph/0507231].
- [50] A. Alboteanu, T. Ohl and R. Ruckl, Phys. Rev. D **76**, 105018 (2007) [arXiv:0707.3595 [hep-ph]].
 - [51] A. Prakash, A. Mitra and P. K. Das, Phys. Rev. D **82**, 055020 (2010) [arXiv:1009.3554 [hep-ph]].
 - [52] P. K. Das, A. Prakash and A. Mitra, Phys. Rev. D **83**, 056002 (2011) [arXiv:1009.3571 [hep-ph]].
 - [53] W. Wang, F. Tian and Z. M. Sheng, arXiv:1105.0252 [hep-ph].
 - [54] A. Alboteanu, T. Ohl and R. Ruckl, Phys. Rev. D **74**, 096004 (2006) [arXiv:hep-ph/0608155].
 - [55] S. K. Garg, T. Shreecharan, P. K. Das, N. G. Deshpande and G. Rajasekaran, JHEP **1107**, 024 (2011) [arXiv:1105.5203 [hep-ph]].
 - [56] A. Alboteanu, T. Ohl and R. Ruckl, eConf **C0705302**, TEV05 (2007) [Acta Phys. Polon. B **38**, 3647 (2007)] [arXiv:0709.2359 [hep-ph]].
 - [57] H. Grosse and Y. Liao, Phys. Rev. D **64**, 115007 (2001) [arXiv:hep-ph/0105090].
 - [58] Y. Liao and C. Dehne, Eur. Phys. J. C **29**, 125 (2003) [arXiv:hep-ph/0211425].
 - [59] J. i. Kamoshita, Eur. Phys. J. C **52**, 451 (2007) [arXiv:hep-ph/0206223].
 - [60] M. Haghighat, N. Okada and A. Stern, Phys. Rev. D **82**, 016007 (2010) [arXiv:1006.1009 [hep-ph]].
 - [61] C. Y. Chen, arXiv:0808.2848 [hep-ph].
 - [62] R. Mertig, M. Bohm and A. Denner, Comput. Phys. Commun. **64**, 345 (1991).
 - [63] J. A. M. Vermaseren, arXiv:math-ph/0010025.
 - [64] K. Nakamura *et al.* [Particle Data Group], J. Phys. G **37**, 075021 (2010).
 - [65] B. Ananthanarayan, S. D. Rindani, R. K. Singh and A. Bartl, Phys. Lett. B **593**, 95 (2004) [Erratum-ibid. B **608**, 274 (2005)] [arXiv:hep-ph/0404106]; B. Ananthanarayan, S. K. Garg, M. Patra and S. D. Rindani, arXiv:1104.3645 [hep-ph].